Pluralities and Plural Logic

Michael Rieppel

On the back cover of Oliver and Smiley's *Plural Logic* there is a photograph of the authors bearing the caption 'Tim and Alex met in the pub and had a pint'.[1] Sentences involving plural terms have until recently not received much attention from logicians, perhaps due to the prevalence of examples with distributive predicates like 'Tim and Alex had a pint', which is equivalent to the conjunctive singular sentence 'Tim had a pint and Alex had a pint'. Things get more interesting if we turn to examples with collective predicates like 'Tim and Alex met in the pub', since this does not predicate having met in the pub of the each author individually, but only of the two taken together. Collective predicates bring out the central question to which this book is addressed: how to treat plural terms like 'Tim and Alex' or 'the authors of *Plural Logic*'.

According to a common "singularist" view, plural terms denote groups, collections, sums, or sets of things, and collective predicates are true of such collections. Oliver and Smiley, by contrast, defend the "pluralist" view that the distinctive character of a plural terms isn't in what's denoted but in the denoting, so to speak: a plural term doesn't denote a plural entity that somehow "collects together" many individuals, but rather plurally denotes those many individuals themselves. The book divides roughly into two parts. The first (chapters 1–10) contains arguments against a number of singularist alternatives to the pluralist view the authors defend, and then goes on to investigate various phenomena involving plural terms as they occur in natural language and informal mathematical contexts. In the second part of the book (chapters 11–14), the distinctive pluralist view that emerges from the first part is incorporated into a formal system, developed in three stages, beginning with a "singular logic," then on to a "mid-plural logic" that introduces plural variables, and finally a "full plural logic" that features bound plural variables. That system is then given application in the form of a novel pluralist axiomatization of set theory. The book is densely argued and refreshingly iconoclastic, combining formal rigor with illuminating discussions of the history of philosophical work on plurals. It recapitulates but also updates work the authors have previously published in a number of joint papers, and will be of interest both to readers looking for an (opinionated) introduction to issues surrounding plurality, and to aficionados interested in a systematic and unified presentation of the distinctive views the authors defend.

To say that this book is simply about plural logic would be to sell it short, however. A significant subsidiary strand of inquiry concerns empty terms. The formal system that the authors develop isn't just a plural logic, but also a (two-valued) free logic that makes room for empty terms, empty domains, and variables whose value may be nothing, or *zilch* as they prefer to put it. Relatedly, they

---

[1] *Plural Logic* by Alex Oliver and Timothy Smiley. Oxford: Oxford University Press, 2013. xiv + 336 pp. £40.00.

defend the idea that there are not just partial functions — ones yielding *zilch* as value for certain arguments — but also *co-partial* functions, which yield an object as value for *zilch* as argument, as well as *multivalued* functions that yield several values for a given argument. The square-root-of function, for instance, is multivalued, yielding both 2 and -2 when applied to 4, and the traditional set-of function is co-partial, mapping the non-self-identical things (aka *zilch*) to the empty set $\{x : x \neq x\}$. The Cantorian set theory the authors develop in the final chapter of the book is not, however, traditional in this sense. Rather than take the membership relation as primitive, they take the set-of function $\{.\}$ as primitive, and construe it in such a way that it yields a set as value when applied to several objects, but none when applied to a single object or to *zilch*. Their set theory in other words does without singletons and the empty set, and is in this way, they argue, closer to Cantor's own conception of sets.

In what follows I will focus on certain points that arise in the first part of the book. Oliver and Smiley's guiding thesis, again, is that plural terms denote plurally, that is, that there is a "semantic relation holding between linguistic expressions ... and things, which is plural in the sense that a particular term may denote several things at once, not just one or perhaps none" (p. 2). This raises the intriguing question whether plural denotation is itself distributive or collective. If a term denotes several things, does it also denote each of those things individually (distributive) or not (collective)? Does 'the authors of *Plural Logic*' denote Oliver and also denote Smiley, or does it only denote the two of them together? Their answer is that there is simply no fact of the matter — a result they call "the indeterminacy of plural denotation." The guiding thought is that plural denotation is whatever relation is involved in plural predication:

(A) $F(a)$ is true iff F is true of the things that a denotes.

Here 'denotes' occurs as part of the plural definite description 'the things that a denotes', so our question about the nature of plural denotation draws in its wake a question about the nature of plural descriptions.

Oliver and Smiley hold that plural descriptions can be interpreted in at least two ways, depending on whether the predicate with which the definite article combines is collective or distributive. In the case of a distributive F, like 'philosophers', 'the F' is interpreted as what they call an *exhaustive* description, and regiment as '$x : Fx$' (drawing on the notation used within the brackets of the set-of functor '$\{.\}$', as in the case of $\{x : x \neq x\}$ above). An exhaustive description denotes the plurality consisting of the various individuals in the extension of F, if there are any, and is empty otherwise. (In order to simplify the exposition, I will occasionally indulge in singular talk of "pluralities" in place of plural talk.) On the other hand, if F is collective, like 'philosophers who met in the pub', 'the F' isn't naturally read as exhaustive, since the extension of F may contain no individual things but only a plurality, meaning that the description would be empty on the exhaustive interpretation. So in the collective case, 'the F' is instead interpreted as what they call a *plurally unique* description, and regiment as '$\iota\mathbf{x}F\mathbf{x}$'. It denotes the unique plurality in the extension of F, if there is one, and is

empty otherwise.[2]

Returning to clause (A), we can use 'denotes$_d$' for the distributive denotation relation and 'denotes$_c$' for the collective one. If the predicate 'a denotes ...' involves the distributive 'denotes$_d$', the description 'the things a denotes' is interpreted as the exhaustive '$x : a$ denotes$_d$ x', and if the predicate involves collective 'denotes$_c$', the description is interpreted as the plurally unique '$\iota\mathbf{x}$ a denotes$_c$ $\mathbf{x}$'. But given the semantics of exhaustive and plurally unique descriptions, the things a denotes$_d$ (i.e. $x : a$ denotes$_d$ x) just *are* the things a denotes$_c$ (i.e. $\iota\mathbf{x}$ a denotes$_c$ $\mathbf{x}$). The difference in the denotation relation is thus "cancelled out" by the difference in the interpretation of the description. Since plural denotation is whatever relation is involved in (A), and since denotation$_d$ and denotation$_c$ will deliver the same truth conditions when plugged into (A), it is simply indeterminate whether plural denotation is collective or distributive.

As this presentation of the argument indicates, Oliver and Smiley hold that plural definite descriptions admit of different interpretations. As a matter of fact, they claim that there are not just two, but three possible interpretations. For just as plurally unique descriptions are the plural analogue of singular descriptions, so exhaustive descriptions have a plural analogue in the form of what they call "plurally exhaustive" descriptions, giving us the following four varieties in all:

$\iota x F x$ *Singular Description:* denotes the unique individual in the extension of F if there is one, and zilch otherwise.

$x : F x$ *Exhaustive Description:* denotes the plurality consisting of all the individuals in the extension of F if there are any, and zilch otherwise.

$\iota\mathbf{x}F\mathbf{x}$ *Plurally Unique Description:* denotes the unique plurality in the extension of F if there is one, and zilch otherwise.

$\mathbf{x} : F\mathbf{x}$ *Plurally Exhaustive Description:* denotes the super-plurality (i.e. plurality of pluralities) consisting of all the pluralities in the extension of F if there are any, and zilch otherwise.

The claim appears to be that the English definite article is four-ways ambiguous. Indeed, Oliver and Smiley seem to suggest that the definite article even has a fifth meaning. Among descriptions they single out functional descriptions like 'the author of *Waverley*' for special treatment. On their view, this description is the result of combining the term 'Waverley' with a functor, 'the author of'. So we here seem to encounter yet another variety of definite article, now one that combines with a relational noun like 'author' to yield an expression 'the author of' that denotes a function.

But is there a need for this proliferation of meanings? Linguistic semanticists commonly adopt a simpler view according to which the definite article has just one meaning. On the functional end of things, the alternative is relatively straightforward: instead of parsing 'the author of *Waverley*' as having the structure [[the author of][*Waverley*]] we parse it as [the [author of [*Waverley*]]], restoring a syntax on which the article combines with a predicate to form a term, as usual. The more interesting divergence emerges in the treatment of plural descriptions. Let me therefore briefly put in place

---

[2]Oliver and Smiley use italics for singular variables, i.e. ones taking a single individual as value, and bold for plural variables, i.e. ones taking a plurality (one or more things) as value.

some of the theoretical background on which this alternative account of plural descriptions rests.

Ontologically, the thought is that the domain of individuals contains both atoms and all the "sums" that can be generated from those atoms. Thus a domain containing the atoms, or singular individuals, a, b, and c will also contain the sums, or plural individuals, $a+b$, $b+c$, $a+c$ and $a+b+c$. The summation operation induces a partial order of inclusion or parthood on the domain:

$$x \leq y \text{ iff } x+y = y$$

(This can alternatively be construed in set theoretic terms: given a set S, the domain of individuals has the structure of the power set of S minus the empty set, with summation the union operation, inclusion the subset relation, and atoms the singleton sets.) Semantically, plural terms have denotations among individuals quite generally, and singular terms have denotations among the atomic individuals — an individual x being atomic just in case $\forall y(y \leq x \rightarrow y = x)$. Plurality is not, however, exclusively a feature of terms. In particular, count nouns come in plural and singular forms, as in 'book' and 'books' or 'author' and 'authors'. Following Link (1983), the plural marker is often understood in terms of a star operator * that functions to close a predicate's extension under the summation operation. So if the extension of 'book' is $\{a,b\}$ the extension of the pluralized 'books' is $\{a,b,a+b\}$. A definite description 'the F', regimented as 'σxFx', is then held to pick out the unique maximal individual in the extension of F, if there is one, and to be empty otherwise. Thus if the extension of 'book' is $\{a\}$, 'the book' will denote (the trivially maximal) a, but if its extension is $\{a,b\}$, 'the book' will be empty; on the other hand, since the extension of the pluralized 'books' will be $\{a,b,a+b\}$, the plural description 'the books' is not empty, but denotes the maximal element $a+b$.[3]

This maximalist semantics for definite descriptions is not just simpler and more unified than Oliver and Smiley's pluralist proposal but also covers much of the same ground. The two proposals coincide when it comes to singular descriptions, and since distributive predicates are such that $\forall x(Fx \leftrightarrow \forall y(y \leq x \rightarrow Fy))$, the maximalist's '$\sigma xFx$' agrees with the pluralist's exhaustive '$x : Fx$' when it comes to plural distributive descriptions. (Modulo the singularist/pluralist disagreement regarding sums versus plural denotation, of course, more on which below.) Furthermore, if we have a collective F that is true of just one plurality, then the maximalist's 'σxFx' again has the same denotation as the pluralist's '$\iota \mathbf{x}F\mathbf{x}$'. The maximalist treatment would thus also serve to vindicate the indeterminacy of plural denotation discussed earlier: maximality together with the contrast between collective and distributive denotation achieves a similar "canceling out" effect as does the distinction between exhaustive and plurally unique descriptions.

Oliver and Smiley recognize that the maximalist approach represents a popular alternative to their own pluralist theory of descriptions, but argue that it ultimately fails. The plausibility of the maximalist view, they argue, rests on a poor diet of examples, and evaporates once we consider a wider range of cases involving collective predicates. Collective descriptions (i.e. ones embedding

---

[3]The maximalist treatment of definite descriptions goes back to Sharvy (1980) and Link (1983). Link (1983) pioneered the appeal to sums in the semantics of plurals. Lasersohn (2011) and Nouwen (2014) provide useful overviews of recent work on plurality by semanticists.

collective predicates) have not received much attention in the literature, I suspect, because the predicates in definite descriptions are usually count nouns, like 'book', whose plural forms are distributive. But as Oliver and Smiley rightly note, it's easy to construct collective examples by modifying the head noun with a relative clause that involves a collective predicate, one of their examples being 'the logicians who wrote *Principia*'. The embedded predicate is collective: it is true of Whitehead and Russell, but not true of either of them individually since, though they were individually logicians, neither of them individually wrote *Principia*. This is a case where the predicate has exactly one sum in its extension, so the maximalist is in the clear. But problems arise when we consider cases where the embedded relative clause applies to more than one sum.

Suppose (to use another of Oliver and Smiley's examples) that three English boys, a, b, and c, jointly solved a puzzle and that three French boys, e, f, and g, also jointly solved the puzzle. Now what does the description 'the boys who jointly solved the puzzle' denote? It looks like the predicate 'boys who jointly solved the puzzle' has the extension $\{a+b+c, e+f+g\}$ which consists of two sums, neither of which is maximal. The description is thus predicted to be empty on the maximalist view. Oliver and Smiley agree that the description has an interpretation, namely the plurally unique one, on which it is empty. But they are able to point to the plurally exhaustive interpretation as one where the description is not empty, but is rather interpreted as denoting a superplurality. Since there are sentences involving this description that sound acceptable, such as 'the boys who jointly solved the puzzle are clever', a non-empty reading does seem available. This is the first problem the maximalist encounters.

Now consider a second, slightly different scenario. Suppose that a and b jointly solved the puzzle using an old method, that a, b and c later jointly solved the puzzle using a new method, and that no one else solved it. The extension of 'boys who jointly solved the puzzle' is thus $\{a+b, a+b+c\}$, so the maximalist predicts that the description 'the boys who jointly solved the puzzle' should denote the maximal sum $a+b+c$. Oliver and Smiley object that, if this were so, the answer to the question 'did the boys who jointly solved the puzzle do so by a new method?' should be a straightforward 'yes'. They find this "quite implausible" since "there is no reason to favour the collaboration between [a, b, and c] over the collaboration between [a and b]" (p. 136). Their own proposal, on the other hand, delivers the desired result. The only non-empty reading of the description that they predict is the plurally exhaustive one, and on it the question would presumably not have a straightforward 'yes' answer, since it isn't the case that both of the relevant pluralities are in the extension of 'solved the puzzle by a new method'.

The objections Oliver and Smiley raise against the maximalist view strike me as important. I am, however, less convinced that the maximalist is bereft of any response. As noted above, many semanticists countenance a pluralizing star operator that functions to close a predicate's extension under the summation operation. The maximalist could perhaps appeal to it to solve the first problem. The suggestion would be that the star operator can be applied to the predicate 'boys who jointly solved the puzzle', with the result that the maximal sum $a+b+c+e+f+g$ would now be included in the predicate's extension, and thus become available to serve as the denotation of the description.

The availability of a non-empty reading for the description 'the boys who jointly solved the puzzle' could thus be secured by appeal to an operation on the embedded predicate rather than by appeal to a special plurally exhaustive meaning for the definite article.

Though this strategy would let the maximalist secure a non-empty reading for the description in the first scenario, it does not deliver the superplural denotation that Oliver and Smiley's plurally exhaustive description is predicted to have. This brings us to the second objection, since a superplural reading is what seems to be needed to avoid the straightforward 'yes' answer to the question 'did the boys who jointly solved the puzzle do so by a new method?' Here too there may be alternatives, however.

One popular approach to broadly superplural phenomena due to Gillon (1987) and further developed by Schwarzschild (1996) appeals to contextually provided "covers" of a sum.[4] To take one of Gillon's examples, consider the sentence 'the men wrote musicals' in a context where the only salient men are Rodgers, Hammerstein, and Hart. By way of background, suppose that Rodgers and Hammerstein collaborated on writing musicals and that Rodgers and Hart also collaborated, but that the three never collaborated on a musical together and none of them ever wrote one on their own. The sentence is true, but how to account for that? On the maximalist view 'the men' denotes the sum $s + n + t$ (abbreviating by the last letters of their names); on Oliver and Smiley's view, the predicted reading (in view of the distributivity of 'men') is the exhaustive $x : man(x)$, which denotes the corresponding plurality. If the predicate 'wrote musicals' is read distributively, the sentence will be false, since none of the three men individually wrote a musical. But neither is the sentence true on a collective reading of 'wrote musicals', since the three men didn't write a musical together (i.e. the sum $s + n + t$ denoted by 'the men' isn't in the extension of the predicate on its collective reading, only the sums $s + n$ and $s + t$ are).

To resolve puzzles like this, the covers-based proposal postulates a distributivity operator ^{D} which attaches to a predicate and alters it so that the predicate gets applied to the elements of a contextually provided cover of the sum denoted by its argument term, rather than to that sum directly, that is, so that '$^D F$' denotes $\lambda x[\forall y(y \in C_x \to F(y))]$ rather than $\lambda x[Fx]$. A cover C of a sum s is a set of sums such that the summation of all the sums in C yields s, or again, such that the maximal element in C^* is s. There are various covers of the sum $s + n + t$, but in the provided context the salient one is $\{s + n, s + t\}$. The sentence 'the men ^{D}wrote musicals' thus receives the following truth conditions:

$$\forall y(y \in \{s + n, s + t\} \to WroteMusicals(y))$$

The sentence is thus true, since Rodgers and Hammerstein wrote musicals and Rodgers and Hart did too.

Notice that a story along these lines appears to be necessary even on Oliver and Smiley's account. They cannot appeal to a plurally exhaustive reading of 'the men' on which it denotes a

---

[4]Schwarzschild (1996) does his semantics in terms of sets rather than sums, but he doesn't regard the difference as important: "our union theory is essentially a set theoretic version of Link's (1983) interpretations of plural noun phrases" (18). I'll here, following Nouwen (2014), describe the proposal in terms of sums.

superplurality corresponding to the cover $\{s+n, s+t\}$, since that would require that the description embed a collective predicate that is true of just those pluralities. The distributive 'men' does not fit this bill, however, since its extension is $\{s, n, t, s+t, s+n, t+n, s+n+t\}$. So even if we were to admit plurally exhaustive descriptions, that won't yet serve to cover all the superplural phenomena we find in English.

At any rate, my suggestion is that the maximalists could appeal to the covers-based proposal to yield the desired results in Oliver and Smiley's second scenario. Recall that the maximalist predicts that 'the boys who jointly solved the puzzle' denotes the sum $a+b+c$. The worry was that this means that the question 'did the boys who jointly solved the puzzle do so by a new method?' has a straightforward 'yes' answer. The maximalist can avoid this result by holding that the partially elided predicate 'solved the puzzle by a new method' can be interpreted relative to the contextually salient cover $\{a+b, a+b+c\}$ of the sum $a+b+c$ denoted by the description. Since it is not the case that every sum in this cover falls within the extension of 'solved the puzzle by a new method' ($a+b$ doesn't), the maximalist no longer predicts the 'yes' answer. The desired result is again achieved by appeal to an operation on a predicate, rather than by appeal to a plurally exhaustive reading of the description. So although I agree that collective descriptions merit greater attention than they have often received in the literature, I do not think Oliver and Smiley's arguments succeed in definitively establishing that the maximalist account of needs to be abandoned in favor of the four-way ambiguity they favor.

The primary objection Oliver and Smiley raise against the practice of semanticists, however, doesn't concern their treatment of descriptions, but their penchant for "changing the subject" from plural talk of many things to singular talk of sums, sets, or other objects that somehow "collect together" those many things. Changing the subject, Oliver and Smiley argue, "is simply not on" (p. 42). They seek to establish this by generalizing an objection Boolos (1984) raised with respect to sets. The strategy is to find a sentence involving plurals that is true, but whose translation into the singularist's idiom of sets, sums, or any other kind of "collection" is false.

Oliver and Smiley's argument proceeds as follows. Take the collection of Whitehead and Russell. It is not a constituent of itself, since only Whitehead and Russell are (individually) constituents of it. Notice that the constituent-of relation at issue here is not the inclusion relation $\leq$ from earlier: the inclusion relation is reflexive, so the collection of Whitehead and Russell is, like everything, included in itself. The constituent-of relation is rather the relation $<$ between a collection and the atomic elements it includes.[5] Given that the collection of Whitehead and Russell is not a constituent of itself, it seems to follow that the collection of Whitehead and Russell is one of the things that are not constituents of themselves (the is-one-of relation being the pluralist's analogue of the constituent-of relation). But upon translation into collectivese, this becomes 'the collection of Whitehead and Russell is a constituent of the collection of things that are not constituents of themselves'. If this sentence were true, there would have to be a collection that has as constituents

---

[5]That is, $x < y$ iff $Atom(x) \wedge x \leq y$. Note that in view of the reflexivity of $\leq$, this means that the predicate 'x is not a constituent of x' is just the predicate '$\neg Atom(x)$'.

all and only the things that aren't constituents of themselves:

$$\exists x \forall y (y < x \leftrightarrow \neg (y < y))$$

Since there is no such collection, the collectivese translation is false, meaning that it does not adequately render the meaning of the true plural sentence it was meant to translate. So changing the subject is "simply not on."

At least two kinds of responses come to mind, however. One option would be to go type theoretic. A collection, the suggestion would run, is always of a higher type than the things that are its constituents, i.e. the things that are construed as "atoms" relative to the collection. The existential quantifier $\exists x$ in the formula above should, in other words, be construed as ranging over things one level higher in the hierarchy from what the succeeding universal $\forall y$ ranges over. Differently put, the "things" at issue in 'the collection of things that are not constituents of themselves' have to be understood as things one level below the collection the description is meant to pick out. Link offers a response along these lines when confronted with problems in the vicinity. He holds that the notion of a sum, or plural object, is "an *inherently relative concept* ... you have to tell me first what 'regular' things you are prepared to include in your domain of discourse, and then it's easy to say what the plural things are in your ontology" (Link, 1998, p. 326).

Oliver and Smiley might reject this kind of response on the grounds that the type theoretic view prohibits us from quantifying over all things at all levels of the hierarchy at once, whereas the topic neutrality of logic must allow for quantification over everything there is (on this see their §11.1). But this demand is one that a friend of collections might reject. Furthermore, the objection could prompt a *tu quoque* response. In the course of developing their semantics, Oliver and Smiley propose that functors and predicates receive functions and relations as their semantic values, where these are explicitly *not* to be construed as entities that could fall within the range of first order quantifiers. "Like Frege's Begriffe, they are not objects at all, and therefore are not values of first-order variables" (p. 195). It thus looks like they don't allow for quantification over quite everything in their ontology either. But maybe that's unfair: quantification over everything, it will be protested, means quantification over all *objects*, and although collections are supposed to be objects of some sort, Oliver and Smiley's Fregean functions and relations are not.

This brings us to a second response. In what sense, exactly, are collections, or sums, supposed to be "objects"? One popular notion of object, deriving from Frege, has it that objects are whatever can be referred to by singular terms. Insofar as e.g. 'the collection of Whitehead and Russell' is a singular term that denotes a collection, collections count as objects. But something has gone awry here. After all, collections, or sums, were precisely supposed to be what plural terms, rather than singular ones, denote. Singular reference to collections by means of descriptions of the form 'the collection of ...' is thus aberrant, and should perhaps just be rejected. Strictly speaking, "collections" can only be talked about using plurals. Even the use of count nouns like 'collection' or 'sum' is apt to mislead. It is not meant to be taken too literally — it's the kind of talk one engages in when

relying on a reader who does not begrudge a pinch of salt. [6]

What makes sums or other collections attractive in the study of plurals isn't their ontological status as objects, but the kind of structure that a theory based on them has. Semanticists aren't, for example, overly concerned about the choice between sets and sums because the same kind of lattice structure can be generated in terms of either. (On this, see e.g. Link (1998, §2.3), Lasersohn (2011, §2.1), and the quote from Schwarzschild (1996) in n4 above.) Once we look at the matter structurally, what is remarkable is the extent of agreement between Oliver and Smiley's plural logic and the kind of view one commonly finds in the semantics literature. Both countenance an inclusion relation $\leq$ and an operation of summation ("plural union" in Oliver and Smiley's terms) related to the inclusion relation in the manner explained above. And both countenance a predicate of atomicity ("singularity" in Oliver and Smiley's terms) defined to hold of a just in case $\forall x(x \leq a \rightarrow x = a)$. One difference between the two treatments is that whereas semanticists use only one class of variables, Oliver and Smiley use two classes of variables (though only one class of constants): italicized singular variables that are bound by singular quantifiers and range over objects, and bold plural variables that are bound by plural quantifiers. But here too the difference may be only skin deep. For as Oliver and Smiley acknowledge:

> "A variant of full plural logic could do without singular variables in favour of plural ones alone .... $S(a)$ [i.e. singularity] is definable as $\forall \mathbf{x}(\mathbf{x} \leq a \rightarrow \mathbf{x} = a)$ where $\mathbf{x}$ is the first plural variable not free in a. We can then go on to define singular quantification in terms of plural quantification by using S with its new plural definition to limit the value of a plural variable to at most one thing — $\forall x A(x)$ can be defined as $\forall \mathbf{x}(S\mathbf{x} \rightarrow A(\mathbf{x}))$" (p. 237).

This suggests that the single class of variables invoked by friends of sums should perhaps just be construed as notational variants of Oliver and Smiley's plural variables.

Is there nevertheless a deeper disagreement? One point that remains is that semanticists are drawn to talk of plural *individuals* or plural *objects* (in contrast to singular or atomic individuals), whereas Oliver and Smiley resolutely oppose it. What should we make of this? If by objects we mean what singular terms denote, then talk of plural objects is, again, at best misleading, since "plural objects" are supposed to be what plural terms, rather than singular ones, denote. But despite the formidable pedigree that this notion of object enjoys, it isn't the only one available. Although pluralities aren't denoted by singular terms, there are other features that they do share with the objects singular terms denote. Pluralities, for example, fall into the extensions of predicates, including, notably, collective ones like 'met in the pub'. To put it in less loaded terms: the predicate 'met in the pub' holds of Oliver and Smiley "taken together," not of either of them individually. Furthermore, if we accept the theory of functions which Oliver and Smiley advocate, then pluralities are both arguments to, and values of, functions, including, importantly, the valuation function that assigns values

---

[6] A variant of this strategy would be to regard collection talk as legitimate, but to construe descriptions of the form 'the collection of Fs' as what Oliver and Smiley call "pseudo-singular terms": terms which, though grammatically singular, are semantically plural (p. 273ff). 'The collection of Fs' would then just be a pleonastic variant of 'the Fs'.

to plural variables. This suggests that a coherent but wider notion of object might be available: one according to which objects are things that (i) are denoted by terms (understood to cover singular and plural), (ii) fall into the extensions of predicates, and (iii) are capable of being arguments to and values of functions. So understood, Oliver and Smiley would count as an object, albeit a plural one — the label "plural object" is just meant to register certain important features they share with objects more standardly conceived, not to suggest that they can be referred to using a singular term like 'the collection of Oliver and Smiley'. Perhaps something along these lines is also involved Russell's talk of the "unity" had by a "class as many" in the following remarks from *Principles of Mathematics*, to which Oliver and Smiley frequently recur:

> The distinction of a class as many from a class as a whole is often made by language: space and points, time and instants, the army and the soldiers, the navy and the sailors, the Cabinet and the Cabinet Ministers, all illustrate the distinction .... In a class as many, the component terms, though they have some kind of unity, have less than is required for a whole. They have, in fact, just so much unity as is required to make them many, and not enough to prevent them from remaining many. (Russell, 1903, §70)

I won't attempt to pursue the matter further, however.

The critical points I have raised are not meant to detract from this book's achievements. The discussion of the indeterminacy of plural denotation and the theory of functions the authors develop, as well as the way the latter idea is applied in the context their semantics and the Cantorian set theory they present, struck this reader as particularly interesting and worthy of further philosophical attention. The objections they raise against the maximalist theory of descriptions also merit more detailed scrutiny. And as for their own theory of descriptions, even if it is to be rejected in its application to English (though I don't take myself to have established as much), it remains the case that it succeeds in clearly marking distinctions that are surely useful in certain formal investigations, which needn't be constrained by the expressive resources available in natural language.

References

Boolos, G. (1984). To Be is to Be a Value of a Variable (or to Be Some Values of Some Variables). *The Journal of Philosophy*, 81(8):430–449.

Gillon, B. (1987). The Readings of Plural Noun Phrases in English. *Linguistics and Philosophy*, 10:199–219.

Lasersohn, P. (2011). Mass Nouns and Plurals. In Klaus von Heusinger, Claudia Maienborn, P. P., editor, *Semantics*, volume 2 of *HSK*, pages 1131–1153. de Gruyter.

Link, G. (1983). The Logical Analysis of Plural and Mass Terms: a Lattice Theoretic Approach. In Bäuerle, R., Schwarze, C., and von Stechow, A., editors, *Meaning, Use and Interpretation of Language*, pages 302–323. Walter de Gruyter, Berlin.

Link, G. (1998). *Algebraic Semantics in Language and Philosophy*. CSLI Publications.

Nouwen, R. (2014). Plurality. In Aloni, M. and Dekker, P., editors, *Cambridge Handbook of Semantics*. Cambridge University Press. Forthcoming. Available at: http://semanticsarchive.net/Archive/DJkMTZlZ/nouwen-plurality-cambridge-handbook.pdf.

Russell, B. (1903). *The Principles of Mathematics*. Cambridge University Press.

Schwarzschild, R. (1996). *Pluralities*. Kluwer Academic Publishers.

Sharvy, R. (1980). A More General Theory of Definite Descriptions. *The Philosophical Review*, 89(4):607–624.